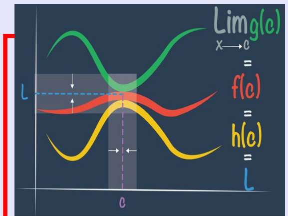
Math 261

Fall 2022

Lecture 6



Evaluate $\lim_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1} = \dfrac{\sqrt{6-2}-2}{\sqrt{3-2}-1} = \dfrac{\sqrt{4}-2}{\sqrt{1}-1} = \dfrac{0}{0}$

I.F.

$$\lim_{x\to 2} \frac{(\sqrt{6-x}-2)(\sqrt{6-x}+2)(\sqrt{3-x}+1)}{(\sqrt{3-x}-1)(\sqrt{6-x}+2)(\sqrt{3-x}+1)}$$

$$= \lim_{x\to 2} \frac{\overset{2-x}{\cancel{(6-x-4)}}(\sqrt{3-x}+1)}{\underset{2-x}{\cancel{(3-x-1)}}(\sqrt{6-x}+2)} = \lim_{x\to 2} \frac{\sqrt{3-x}+1}{\sqrt{6-x}+2}$$

$$= \frac{\sqrt{3-2}+1}{\sqrt{6-2}+2} = \frac{\sqrt{1}+1}{\sqrt{4}+2} = \frac{2}{4}$$

$$= \boxed{\frac{1}{2}}$$

Evaluate

Review

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$$\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \frac{\sqrt[3]{1}-1}{\sqrt{1}-1} = \frac{0}{0} \quad \text{I.F.}$$

$$\lim_{x\to 1} \frac{(\overset{A}{\sqrt[3]{x}} \overset{-B}{-1})(\sqrt{x}+1)(\overset{A^2}{\sqrt[3]{x^2}} \overset{+AB}{+\sqrt[3]{x}} \overset{+B^2}{+1})}{(\sqrt{x}-1)(\sqrt{x}+1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} = \lim_{x\to 1} \frac{\sqrt{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$= \frac{\sqrt{1}+1}{\sqrt[3]{1}+\sqrt[3]{1}+1}$$

$$= \boxed{\frac{2}{3}}$$

Find a & b such that

$$\lim_{x\to 0} \frac{\sqrt{ax+b}-2}{x} = 1$$

$$\lim_{x\to 0} \frac{(\overset{A}{\sqrt{ax+b}} \overset{-B}{-2})(\overset{A}{\sqrt{ax+b}} \overset{+B}{+2})}{x(\sqrt{ax+b}+2)}$$

$$= \lim_{x\to 0} \frac{\overset{A^2}{ax+b} \overset{-B^2}{-4}}{x(\sqrt{ax+b}+2)} = \lim_{x\to 0} \frac{ax+\overset{\to 0}{(b-4)}}{x(\sqrt{ax+b}+2)}$$

$b-4=0$

$\boxed{b=4}$

$$= \lim_{x\to 0} \frac{a\cancel{x}}{\cancel{x}(\sqrt{ax+4}+2)}$$

$$= \lim_{x\to 0} \frac{a}{\sqrt{ax+4}+2}$$

$$= \frac{a}{\sqrt{0+4}+2} = \frac{a}{4}$$

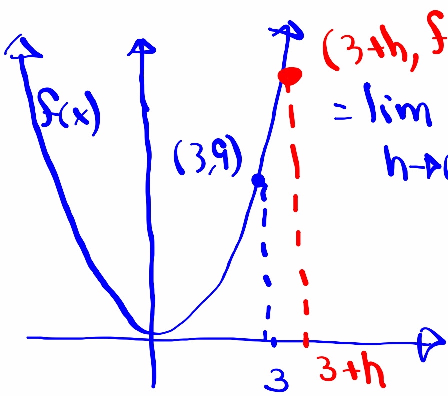
but $\lim_{x\to 0} \frac{\sqrt{ax+b}-2}{x} = 1$

$\frac{a}{4} = 1 \quad \boxed{a=4}$

Given $f(x) = x^2$

$(A+B)^2 = A^2 + 2AB + B^2$

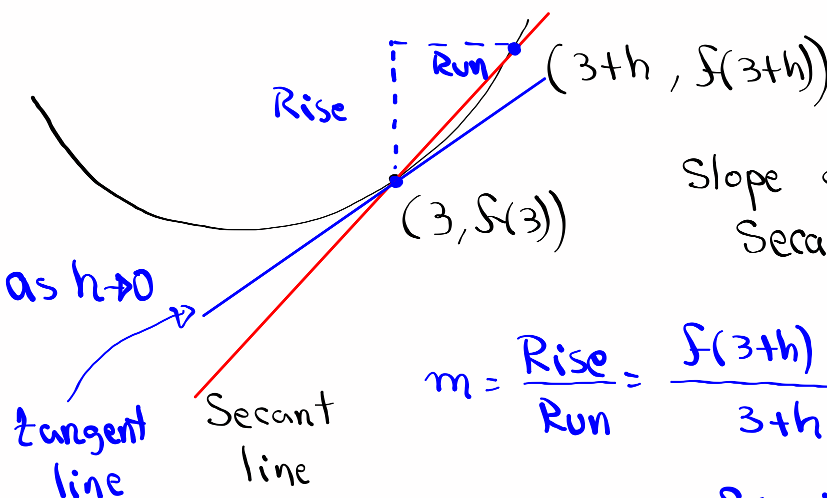
Evaluate $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h} = \lim\limits_{h \to 0} \dfrac{(3+h)^2 - 3^2}{h}$



$= \lim\limits_{h \to 0} \dfrac{9 + 6h + h^2 - 9}{h} = \lim\limits_{h \to 0} \dfrac{6h + h^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{h(6+h)}{h}$

$= \lim\limits_{h \to 0} (6+h) = \boxed{6}$



Slope of this Secant line

$m = \dfrac{\text{Rise}}{\text{Run}} = \dfrac{f(3+h) - f(3)}{3+h-3}$

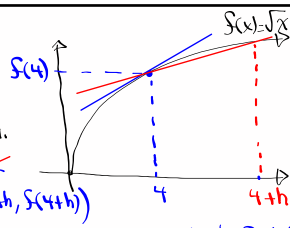
$= \dfrac{f(3+h) - f(3)}{h}$

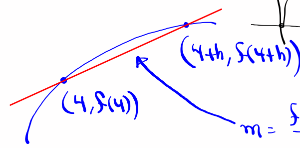
Slope of the tangent line

$\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h} = \boxed{6}$

Consider $f(x)=\sqrt{x}$

Find slope of the tangent line at $x=4$.





Not Scaled

$$m=\frac{f(4+h)-f(4)}{4+h-4}=\frac{f(4+h)-f(4)}{h}$$

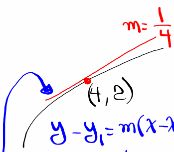
Slope of the tangent line at $x=4$ $=\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$

$$=\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}=\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$$

$$=\lim_{h\to 0}\frac{\overbrace{(\sqrt{4+h}-2)}^{A-B}\overbrace{(\sqrt{4+h}+2)}^{A+B}}{h(\sqrt{4+h}+2)}$$

$$=\lim_{h\to 0}\frac{\cancel{4}+\cancel{h}^{1}-\cancel{4}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{\sqrt{4+0}+2}=\boxed{\frac{1}{4}}$$



$y-y_1=m(x-x_1)$

$y-2=\frac{1}{4}(x-4)$

$\boxed{y=\frac{1}{4}x+1}$

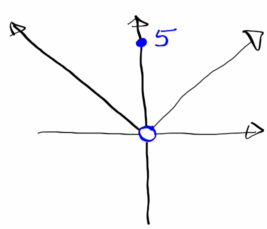
Slope of the tangent line to the graph of $f(x)=\sqrt{x}$ at $x=4$.

$f(x)$ is continuous at $x=a$ if

1) $f(a)$ exists
2) $\lim_{x\to a} f(x)$ exists
3) $\lim_{x\to a} f(x)=f(a)$

Ex:

$$f(x)=\begin{cases}|x| & \text{if } x\neq 0\\ 5 & \text{if } x=0\end{cases}$$



$\lim_{x\to 0^-} f(x)=|0|=0$ $\qquad$ $\lim_{x\to 0^+} f(x)=|0|=0$

$\lim_{x\to 0} f(x)=0$ $\qquad$ $f(0)=5$

$f(x)$ is not continuous at $x=0$

$$f(x) = \begin{cases} \frac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ \frac{1}{2} & \text{if } x = 1 \end{cases}$$

$f(1) = \frac{1}{2}$ ✓

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2-x}{x^2-1} = \lim_{x \to 1^-} \frac{x(x-1)}{(x-1)(x+1)} = \lim_{x \to 1^-} \frac{x}{x+1} = \boxed{\frac{1}{2}}$$

$$\lim_{x \to 1^+} f(x) = \cdots\cdots = \frac{1}{2}$$

So $\lim_{x \to 1} f(x) = \frac{1}{2}$ ✓

$\lim_{x \to 1} f(x) = f(1)$

So $f(x)$ is cont. at $x = 1$

Precise def. of limits:

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$$\lim_{x \to a} f(x) = L$$

Show for $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\lim_{x \to 2} \left(\tfrac{1}{2}x + 3\right) = 4$$ ✓

$f(x) = \frac{1}{2}x + 3$

$a = 2$, $L = 4$

For $\boxed{\varepsilon>0}$, there is a $\delta>0$ such that

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$\left|\frac{1}{2}x+3-4\right|<\varepsilon$ whenever $|x-2|<\delta$

$\left|\frac{1}{2}x-1\right|<\varepsilon$

$\left|\frac{1}{2}(x-2)\right|<\varepsilon$

$\frac{1}{2}|x-2|<\varepsilon$

Multiply by 2

$|x-2|<2\varepsilon$

$\boxed{\text{Pick } \delta=2\varepsilon}$